THE PHASE NOISE SPECTRUM AND STRUCTURE OF PHOTONS?

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ABSTRACT

This is a speculative paper based on the EM evanescent wave model presented in the 2009 IFCS-EFTF conference [1]. It is about photons in free space.

We can measure the spectral line width and frequency of a source of photons. Higher measurement accuracy is obtained by integrating many photons over a sufficiently long time. Then we see a continuous source with an oscillator like spectrum. An individual photon is assumed to have the same spectrum.

The structure of a single photon is proposed to be a cylindrical rod or 'arrow' of EM energy with finite length and diameter. The length is assumed to be the reciprocal of the spectral line width times the velocity of light. The photon has a finite volume in which the energy density is approximately uniform.

The proposed cross-section is a circular radial transverse electric evanescent wave as seen on the non-radiating Goubau single wire transmission line. However for the photon there is no centre conductor to support the photon wave. The radius of the arrow is assumed to be proportional to the square root of the photon frequency as is found for the energy surrounding a Goubau line.

The energy of a photon is its frequency times the Planck constant. The energy density is taken to be approximately constant within the cylindrical shape of the photon. Thus the energy density per Hz (at the peak of the spectrum) is independent of the photon length. It is inversely proportional to the cross-section area.

But thermal noise per Hz per square metre of a surface is assumed to be kT, (Boltzmann's constant times the absolute temperature). Note that this is possibly a new assumption. It is based on observations of thermal noise captured by an antenna of known beam width and capture area.

When the peak energy density of the photon falls below the total (thermal) energy density (kT) we postulate that the photon is no longer stable and will lose its structure and 'evaporate'. It will merge with other adjacent photons to form a continuum of energy, but the continuum will still have the line spectrum of the original photons.

We find that there is a critical frequency, proportional to the square root of temperature, below which photons cannot exist as particles.

A rough estimate of the critical frequency is given. It is based on observed the few available measurements of transverse EM coupling and the Goubau line critical radial distance at various frequencies.

PHOTON MODEL

The proposed model of a photon is a cylindrical arrow with very nearly uniform energy density travelling at the speed of light.

PHOTON SPECTRUM AND LENGTH

Fig. 1 shows the measured spectrum of a source of photons. The spectrum of an individual photon is assumed to be the same. Note that the spectrum has a finite bandwidth. The bandwidth is defined as the speed of light divided by the 'coherence length' of the photon. The coherence length of the photon is taken to be its physical length.

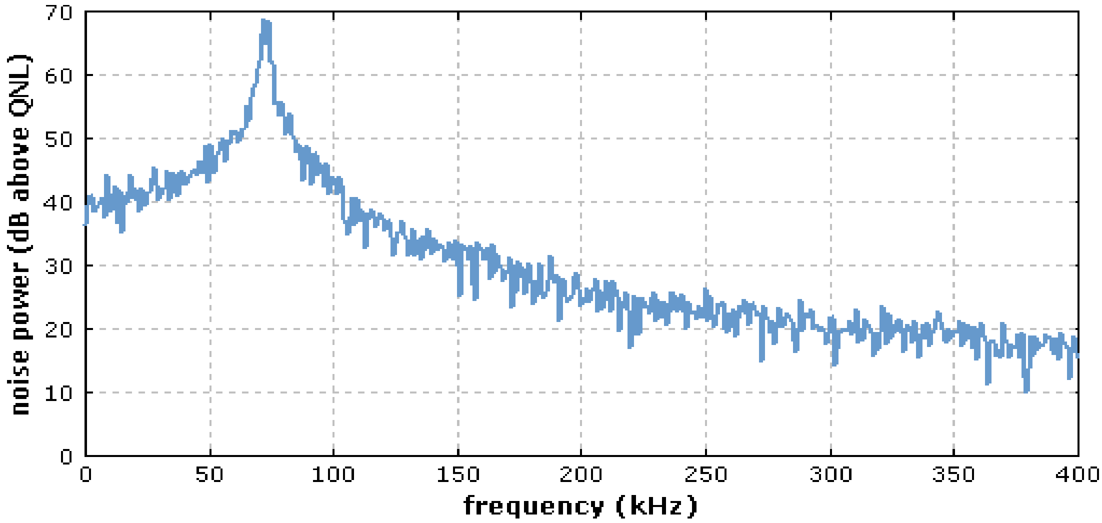

Fig. 1. Intensity noise spectrum of a solid-state laser. Here, the power spectral density relative to that of shot noise is displayed. (from: "Encyclopedia of Laser Physics and Technology" by Dr. Rüdiger Paschotta, RP Photonics published also by Wiley)

Fig. 1 does not show the exact shape of the peak of the spectrum in sufficient detail so that a definitive formula may be derived for it. At first sight it looks the same as the Leeson oscillator model spectrum [2]. Fig. 2 shows the Leeson oscillator model and the first part of the derivation of the spectrum shape.

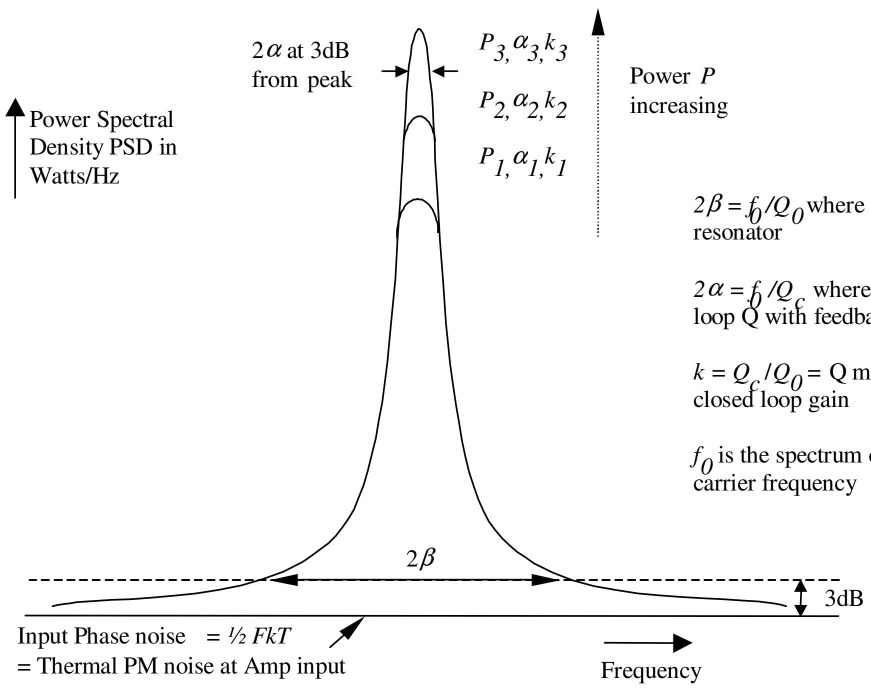

Fig. 2. Leeson Oscillator Model and its Phase Noise Spectrum

But the Leeson model cannot represent the shape of a photon spectrum exactly. The actual spectrum of a photon must reflect the fact that it is a stable packet of energy transported at the speed of light. It does not lose energy. Thus in Leeson oscillator terms its Q must be infinite and its (3dB) bandwidth should be zero.

In the following we propose a cross section energy density function for the photon similar to that found for the Goubau single wire transmission line. It is a 'self-coupled evanescent wave' distribution function. This is one of a class of distribution functions that are 'evanescent' and stable, and so do not decay with time. It can also be considered as a spatial Fourier or Laplace transform. The same shape distribution is proposed in the frequency domain to represent the spectrum shape. This first heuristic guess (hypothesis) is open to future improvement or practical confirmation when better spectrum shape measurements have been made.

SELF-COUPLED DISTRIBUTION FUNCTIONS

The self-coupled distribution functions are proposed for widespread use within electromagnetics and physics. They are applicable to evanescent waves as seen on a Goubau line. They have a 'material' density that is finite at the origin and thereafter decays exponentially to zero with a chosen negative integer n of distance. The tail of the distribution rapidly becomes $1/r^n$ with distance r. The integer n can be made unity to represent a spherically symmetric potential. It can be two to represent field amplitude. And it can be three to represent the density of induced (secondary) source material.

These functions can replace Green's functions with advantage. They are conveniently 're-normalised' and do not have any essential or simple spatial singularities. They are 'meromorphic' functions.

The generating equation is based on the concept of a spherically symmetric particle (as a Higg's particle?) source that by 'induction' 'creates' or 'destroys' its own source material density M_d according to a chosen 'self-coupling' distance ($1/r^n$) function. The self-coupling factor is a. The generating differential equation is shown in (1):

$$\frac{dM_d}{M_d} = -\frac{a_n\,dr}{r^{n+1}} \tag{1}$$

The obvious solution of (1) is:

$$\frac{M_d}{M_{d0}} = e^{-\frac{a_n}{nr^n}} \tag{2}$$

But (2) does not represent observable physics. We use intuition to look for and find a second solution for (1) with the right physical behaviour:

$$\frac{M_d}{M_{d0}} = 1 - e^{-\frac{a_n}{nr^n}} \tag{3}$$

This is used to define the profile of the photon cross-section on the basis that it is the same as the energy distribution of the energy stored around a Goubau single wire transmission line in the following. The energy density can be factorised into two processes. The first is for n =1 and this represents the potential magnitude distribution. The second is for n =2 and this represents the energy amplitude (the square root of the energy density). The third is for n= 3 and then it is the magnitude of the displacement current amplitude density. The shapes of these three functions are shown in Fig, 3.

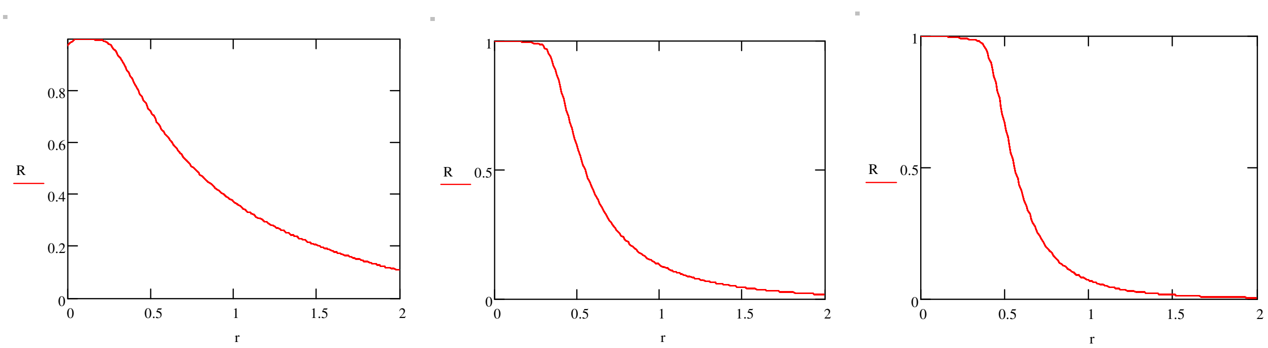

Fig. 3. Plots of radial distribution functions: left is potential magnitude; middle is energy amplitude magnitude; and right is displacement current magnitude.

In these cases a cylindrical coordinate system has to be assumed for the local space in place of the spherical coordinate system assumed in the derivation of (3). The process described in (1) remains the same with *r* representing the cylindrical radius.

SELF-COUPLING AND MAXWELL'S EQUATIONS.

Note that the concept of self-coupling is not included in Maxwell's equations as conventionally stated. These equations do not include sources explicitly. It is only when a spherical coordinate system is imposed that we find that the *div* and *curl* differentials may be integrated to give point source equations with associated Green's functions. Distributed sources are then created artificially from a distribution of point sources. The assumption is that linear superposition applies, which in general is the case. But this assumption appears to rule out any concept of self-coupling (or resonant antenna modes). The solution is to (re-)introduce explicit distributed sources equations, such as Gauss's Law and the Biot-Savart Law, into an extended Maxwell equation set. Historically such equations were originally present in the twenty or so equations that Maxwell set out in his 'treatise on electromagnetism' [6] and [7] but were subsequently missed out after his death by the 'Maxwellians' [5].

Thus the concept of electromagnetic self-coupling can be regarded as a new and arguably far-reaching discovery.

SPREADING FUNCTIONS AS PROCESSES

The 'spreading function' is a separate process from 'self-coupling' as examined above. Essentially it describes the coordinate system that is operative at any point in space surrounding a source. For example for the fields displacement currents and potentials close to a charged wire also carrying a current, cylindrical coordinates should be used. Well away from a finite length wire spherical coordinates should be used. There is a coordinate system transition or 'handover' region that can be estimated well enough for practical engineering purposes as explained in the next section. Spreading functions may conveniently be described by function corresponding to making $n = 0$ in (1). We then have:

$$\frac{dM_d}{M_d} = -\frac{b\,dr}{r} \tag{4}$$

The solution to this is:

$$\frac{M_d}{M_{dr0}} = (\frac{r_0}{r})^b \tag{5}$$

The parameter b is usually an integer. For example the inverse square law in free space is represented by b = 2. The inverse law for the fields around a cylinder or for a surface wave on a lossless surface is represented by n = 1.

These spreading functions have a singularity at the origin. Thus it is always better to use the functions above with n>0 with the parameter a_n suitably chosen.

PHOTON CROSS-SECTION

The proposition is that the cross section of the photon arrow is similar to the energy density cross-section of the Goubau single wire transmission line. The similarity is in the evanescent wave layered energy structure that surrounds the Goubau line as described in [1] and shown in Fig.3. The alternating layers are of two types of energy as can be seen on a shorted or un-terminated wire transmission line. One type is the stored energy associated with the current. The other is the stored energy associated with the voltage or charge. The total energy is the sum of the two types and it is a monotonic function. For the Goubau line and the here is both standing wave energy and travelling wave energy. But the standing wave energy density is far greater than the travelling wave energy.

DOMINANT PROCESS REGIONS AND BOUNDARIES

In numerous places in electromagnetics we observe that processes that are co-located but distributed in a finite volume of space ‘power combine’ the amplitude effects of the processes according to the ‘Root-Sum-of-Squares’ or RSS law. An example of this is the combination of the radiation and loss resistances of small tuned loop antennas [3].

On the assumption that spreading functions, evanescent waves, and many other if not all physical processes obey the RSS law we can define regions in which only one process is dominant. In this region all other processes are suppressed according to the RSS law. Region boundaries are defined to be the lines or surfaces where the strongest two processes have exactly equal effect.

We define a process region as being where only one process need be considered for practical (engineering) predictions. Region boundaries are where two processes have equal effect. This engineering simplification can be considered to be similar to the use of ‘Bode plots’ by control engineers.

The technique is particularly useful for use in the near-field of wire antennas where there practically always is a transition from the cylindrical coordinate system of the wire to the spherical coordinate system of free-space radiation. It is also useful for ground wave propagation over a lossy ground. The transition from the spreading function to the exponential loss function can be defined as a region boundary.

THE GOUBAU LINE MODEL OF A PHOTON

The Goubau single wire transmission line was first published by Georg Goubau in 1950 [4]. Fig. 4 shows the original arrangement that was measured. Although the electromagnetic assumptions and the original explanation of its operation are not correct in every detail, the Goubau line should be regarded as the most significant discovery in electromagnetics in the twentieth century. The reason is that it contradicts the underlying basis of most if not all antenna CAD programmes. These depend on the assertion ‘it is the current that radiates’. Outside the horns the Goubau wire line carries a current with little or no radiation from it. This reveals a fundamental misconception of the way in which (wire) antennas work. Thus in due course most antennas CAD have to be revised to establish the fundamental limits of capability and to improve the accuracy of model predictions. (The theoretical and CAD model predictions of small antennas are currently the most questionable.)

One original error was the assertion that the wire requires a dielectric layer coating in order to suppress the radiation. However 50um thickness of enamel with relative permittivity of about 3 and loss tangent of 8×10^{-3} will be destroyed by powers of little more than a watt. At this author’s suggestion the company Rubytron from Port Chester NY, USA manufactured a low loss Goubau line using bare aluminium wire. The explanation is that the presence of the conductive wire increases the effective permeability of local space around the wire. It forms a ‘local ether’. A displacement current flows in this such that the combination of this displacement current and the real current on the wire does not radiate. There is thus an evanescent wave surrounding the line.

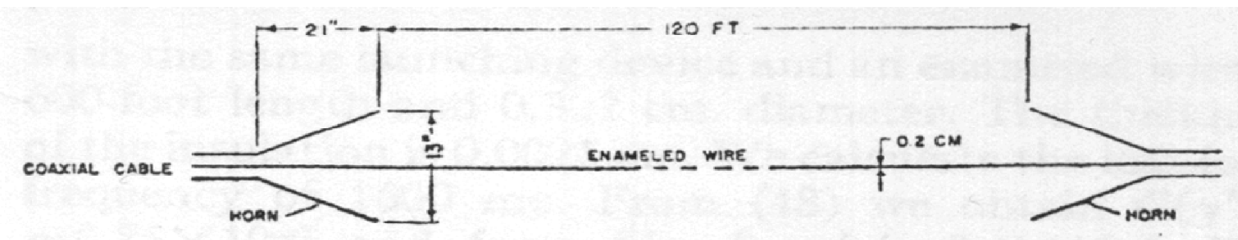

FIG. 11. Sketch of a surface wave transmission line.

Fig. 4 Original Goubau single wire non-radiating transmission line arrangement. From “Surface Waves and Their Application to Transmission Lines”, by Georg Goubau. J.A.P.,Vol.21, Nov., 1950, pp1119-1128 .

In [1] the evanescent wave, on a conducting surface or on a Goubau line, was shown to be layered with alternating layers of different types of energy. The radial distance r_h *(=* a_n in (3)*)* to which the energy extends is found by measurement to be proportional to the square root of frequency. This accounts for the increased number of layers as the frequency increases. Fig. 4 to Fig. 6 illustrates one set of layers and how these are contained within the horn diameter.

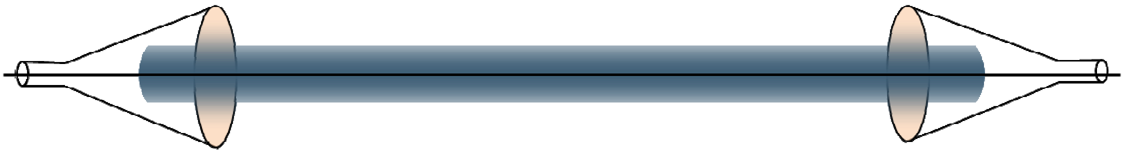

Fig. 4. Layers of lowered spatial impedance around a Goubau line holding field or displacement current layers. Horn Diameter = 0.1λ at ~20MHz, or 1.5m. Layers are λ/2 apart. Minimum horn size is proportional to √f

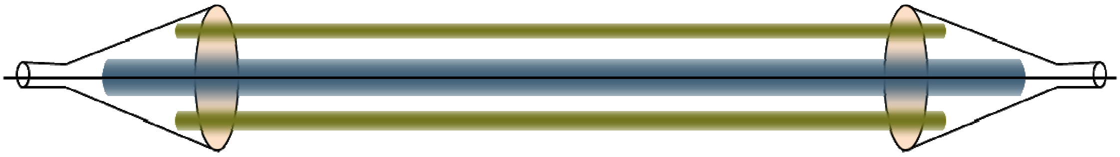

Fig. 5. Layers of lowered spatial impedance around a Goubau line holding field or displacement current layers. Horn Diameter = 1.3λ at ~1.3GHz or 30cm. Layers are λ/2 apart. Minimum horn size is proportional to √f

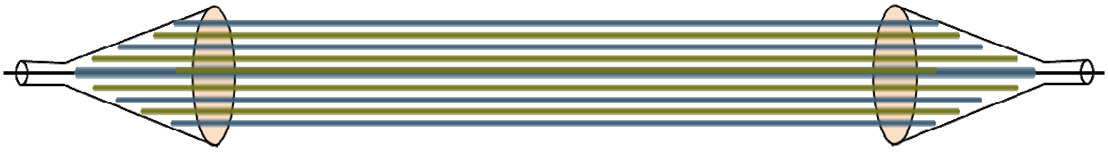

Fig. 6. Layers of lowered spatial impedance around a Goubau line holding field or displacement current layers. Horn Diameter = 5λ at ~13GHz or 11.5cm. Layers are λ/2 apart. Minimum horn size is proportional to √f

Fig.7 shows the interlaced layers of two types of energy. One type is analogous to the magnetic energy seen around the current antinodes of a standing wave on a transmission line. The other is analogous to the electrostatic energy seen around the voltage antinodes on a transmission line. The right hand picture is for the total energy of the two layers added together.

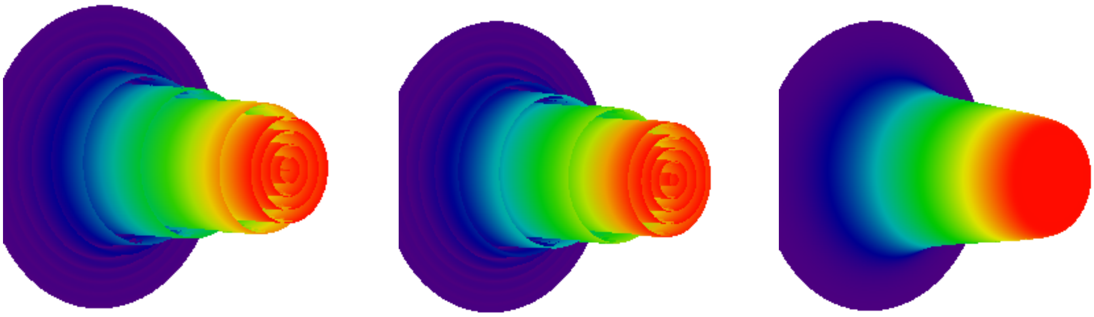

Fig. 7. Representation of Goubau line or photon layer magnitudes. Red is maximum and blue is minimum. Left is for 'sine' layers. Middle is for 'cosine' layer. Right is magnitude of total energy.

Note that on a Goubau line the electric field (E or D) is radial (to infinity) and the magnetic field (H) is solenoidal in circular loops around and perpendicular to the wire.

PHOTON STABILITY IN THERMAL NOISE

As a possibly new proposition, for free space with a temperature of T the thermal noise flux in watts per square metre per Hz is assumed to be kT, where k is Boltzmann's constant. This is the Johnson-Nyquist thermal noise law applied to free space.

The Stefan-Boltzmann noise energy flux spectrum is defined on the surface of a hot object. At a distance the flux is attenuated by an appropriate spreading function. The spectrum remains the same but its intensity is diminished. Thus Stefan-Boltzmann Law does not define the noise flux in free space in terms of the temperature of space. The Johnson-Nyquist law should be used.

The thesis of this paper is that as the photon frequency is lowered, the photon peak flux per Hz eventually drops below the noise flux level. At that point the photon will no longer be stable and its energy will start to spread. The photon stability region will shrink to zero and its structure as an individual photon will be lost. The photon particle 'evaporates' into a 'gas'.

ESTIMATION OF FREE SPACE PHOTON STABILITY LIMITING FREQUENCY

The photon of frequency f is assumed to be a cylindrical arrow of radius r and length l and cross section area = πr^2.

The energy of a photon of frequency f is given by $E = h\,f = U_p \times 2\delta f \times l \times Area$. But $2\delta f \times l = c$. So we have $Up = h \times f/(c \times Area)$. Note that this is independent of photon length.

But r_h is the Goubau distance. The best estimate so far from Goubau line horn cut-off frequency measurements is

$$r_h = (14MHz/f)^{1/2}\ (\pm 10\%) \tag{6}$$

So that the $Area = \pi\, r^2 = \pi \times 1.4 \times 10^7 /f$.

$$\text{And then} \qquad U_p = hf^2/(\pi c \times 1.4 \times 10^7) \tag{7}$$

where: δf is the photon line-width; U_p (W m^{-2} Hz^{-1}) is the photon radiation intensity per Hz; Planck's constant h= 6.63 × 10^{-34} m^2 kg / s; Boltzmann's constant k= 1.38× 10^{-33} W/K; and velocity of light c =3× 10^8 m/s.

We suppose that a photon is no longer stable if the (peak) radiation intensity per Hz is equal to the Johnson-Nyquist thermal noise per Hz, $U_J = kT$. For photon stability we then have:

$$U_p \geq U_J \text{ or } hf^2/(\pi c\, f_c) \geq kT. \text{ And this gives} \qquad f^2 \geq (kT/h)\, \pi c\, f_c \tag{8}$$

where we also have: f_c ~ 14MHz (±20%); and U_p (W m^{-2} Hz^{-1}) is the photon radiation intensity

$$\text{The photon stability criterion therefore is:} \qquad f \geq T^{1/2} \times 16.6 \text{ THz} \tag{9}$$

$$\text{At room temperature 300K we have} \qquad f \geq 285\text{THz} \qquad \text{or } \lambda = 1.05\mu\text{m} \tag{10}$$

$$\text{At free space temperature of 3K we have} \qquad \text{f} \geq 28.5\text{THz} \qquad \text{or } \lambda = 10.5\mu\text{m} \tag{11}$$

Because of similarity to the 'capture effect' seen in injection locked oscillators we expect a photon 'condensation-evaporation' difference of +3 to –6dB in the frequency [5] and [6].

CONCLUSIONS

A photon in free space is a cylindrical 'arrow', travelling at the speed of light, of radius = $(f_c/f)^{1/2}$ where f_c obtained from Goubau single wire non-radiating transmission line and surface wave measurements as ~ 14MHz. The photon length is $c/2\delta f$ where $2\delta f$ is the photon line bandwidth.

The cross–section of the photon is similar to the distribution of energy that surrounds the Goubau line. It is a number of interlaced layers of two complementary types e.g. $Cos(kr)$ and $Sin(kr)$ of radial distance r. The edge of the energy distribution of the photon is significantly sharper than for the Goubau line. It probably is this that makes the photon stable and non-dissipative. As a consequence the line width in the frequency domain appears to be sharper. It is flatter with sharper sides than the Leeson oscillator model.

The line width is finite in spite of the infinite predicted Q of a lossless store of energy. We can assign a (measured) bandwidth 2δf and hence an effective Q_e.

A photon of frequency f is assumed to be stable if its peak energy density per Hz is greater than the thermal noise flux per Hz in space, $U_{nf} = kT$. But the total energy in a photon is $E = hf$. The photon length is c/2δf where $2\delta f$ is the photon line bandwidth Thus the photon cross-section area is $\pi r^2 = \pi f_c/f$. We then find that the peak photon power density flow per Hz is $U_{ph} = 2hf^2/c^2$.

The photon stability criterion therefore is: $f \geq T^{1/2} \times 16.6$ THz.

At room temperature 300K we have $f \geq 285$THz or $\lambda \leq 1.05\mu$m. From (6) this corresponds to a photon diameter of 0.22mm

At free space temperature of 3K we have $f \geq 28.5$THz or $\lambda \leq 10.5\mu$m. This corresponds to a photon diameter of 0.70mm.

Because of the ‘capture effect’ seen in injection locked oscillators we expect a photon ‘condensation-evaporation’ difference of +3 to –6dB in the frequency [5] and [6].

These predictions need to be confirmed or otherwise by experiment. This will not be easy. If photon detection is used then the detector itself could assemble a photon packet out of a continuum of electromagnetic energy? This could happen at frequencies considerably lower than the predicted free space photon stability frequency.

A better approach would be to make a linear rf amplifier capable of working to over 300THz. It would have to have a noise figure better than 3dB to be able to detect continuous radiation at suitably low level.

A small Goubau horn having a line truncated at the horn mouth could prove a suitable photon receiving antenna. The heuristic Goubau horn radius formula (6) gives the likely minimum horn radius (= photon radius) for a 300THz photon of 1µm wavelength as $r_h \geq (14\times10^6/3\times10^{14})^{1/2} =$ 0.216mm ≡ 216 wavelengths. This is a horn diameter of greater than about 0.432mm or 432 wavelengths. In practice a Goubau horn antenna diameter two to three times larger would allow for some latitude in photon position and direction.

It would also be interesting to have an array of closely spaced small horns with separate receivers. An intense beam of photons would tend to coalesce into an un-quantised stream. The signals from the receivers would then look highly correlated. For a very weak beam the photons should become discrete entities in space. The received signals should then become uncorrelated.

REFERENCES

[1] M. J. Underhill, “More fundamental instabilities in oscillators?”, *15th EFTF’09:* IEEE-FCC’09, Besancon, France, 20-24 April 2010, pp 334-337.

[2] D. B. Leeson, “A Simple model of feedback oscillator noise spectrum”, Proc IEEE, 54, No. 2, 1966, pp. 329-330.

[3] M. J. Underhill and M. Harper, “Small Antenna Input Impedances that Contradict the Chu-Wheeler Q Criterion”, Electronics Letters, 23rd May 2003, Vol. 39, No. 11.

[4] Georg Goubau, “Surface Waves and Their Application to Transmission Lines”, J.A.P., Vol. 21, Nov. 1950, pp , 1119 – 1128.

[5] Bruce J. Hunt. *The Maxwellians*, Cornell University Press 1991, ISBN 0-8014- 8234-8

[6] J. Clerk Maxwell, “A Treatise on Electricity and Magnetism, 3rd ed.”, Vol. I, Dover reprint, ISBN 0-486-60636-8.

[7] J. Clerk Maxwell, “A Treatise on Electricity and Magnetism, 3rd ed.”, Vol. II, OUP reprint, ISBN 0-19-850374-1.

[8] M. J. Underhill, N. Siripon, and N. Christophorou, “Determination of the capture range and locking range of injection locked oscillators”, 15th EFTF, Neuchatel, Switzerland, March 2001, pp 232-236.

[9] M. J. Underhill, “Fundamental limitations of oscillator performance”, IEE Conf. Pub. 303, Frequency control and synthesis, 1989, pp 18-31.